An Extended VIKOR Method for Multi Attribute Decision Making Under Interval Type-2 Fuzzy Sets Environment

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Abstract—An extended VIKOR method is proposed for fuzzy multi attribute decision making based on interval type-2 fuzzy sets. A new signed area function with trapezoidal fuzzy number is firstly defined and a new ranking score method is presented consequently. Then, based on the ranking score method, interval type-2 trapezoidal fuzzy number matrix can be converted to ranking score value matrix. Finally, the VIKOR method can be extended to deal with multi attribute decision making problems under interval type-2 fuzzy sets environment. The effectiveness of the proposed method is illustrated by a numerical example.

I. INTRODUCTION

Zadeh[1] proposed the definition of type-2 fuzzy set(T2FS) in 1975 to handle higher levels of uncertain information. Different from T1FS, T2FS contains two membership, and the secondary membership is also a fuzzy set, which is more effective to express vague and imprecise information in real-world applications[2]. Recently, interval type-2 fuzzy sets (IT2FSs) have been extensively applied in fuzzy multiple attributes decision making areas. Chen [3] presented a fuzzy ranking method based on IT2FSs and obtained the final rank order of alternatives by TOPSIS method. Wang [4] proposed a new ranking value method to convert the interval type-2 trapezoidal fuzzy numbers(IT2TrFNs) to real numbers and obtained the final rank order based on the integrated ranking value. In [5], the notion of trapezoidal interval type-2 fuzzy soft sets was presented based on interval type-2 trapezoidal fuzzy sets (IT2TrFSs) and soft sets. Chen [6] proposed a new ranking value method to handle fuzzy multiple attributes group decision-making problem where the attribute value takes the form of IT2FSs. Hu [7] presented a new possibility degree of IT2TrFNs to get the rank order of all alternatives. Chen[8] proposed a hybrid averaging approach based on signed distances, and combined with the ELECTRE method to deal with multiple attributes decision-making (MADM) problems. In [9], a new fuzzy ranking method of IT2FS is proposed based on $\alpha$-cuts, which can deal with MADM problems under IT2FS environment and consider decision maker’s attitude about risks simultaneously.Qin[10] proposed a combined ranking value method based on the forward three kinds of ranking value formulas, and a combined ranking value is obtained by the proposed method to select the ideal alternative. Qin[11] presented a extended TOPSIS method based on $\alpha$-cuts and extension principle. In [12], an extended TODIM method is proposed based on IT2FSs to deal with the problem of green supplier selection for automobile manufacturers. In [13], an extended QUALIFLEX method is presented based on IT2FSs and signed distance. Chen[14] combined LINMAP method with Minkowski distance to solve supplier selection problem and compared the results with Chen’s method in [15].

Different from the above methods, VIKOR method can obtain a set of compromise solutions when the criterion conflicts with each other. The compromise solution provides a balance between the maximum group utility value and the minimum individual regret value. Recently, VIKOR method has been widely used to deal with different types of MADM problems[16-19]. In this paper, we propose an extended VIKOR method with ranking score value of IT2TrFNs to solve multi attribute decision making problems under IT2FSs environment.

II. PRELIMINARIES

A. Interval type-2 trapezoidal fuzzy sets

Definition 1[8]: An interval type-2 trapezoidal fuzzy set $A$ can be defined as:

$$A = \left[ A^-, A^+ \right] = \left[ \left[ a^-_1, a^-_2, a^-_3, a^-_4, h^-_1, h^-_2 \right], \left[ a^+_1, a^+_2, a^+_3, a^+_4, h^+_1, h^+_2 \right] \right]$$

where $0 \leq a^-_1 \leq a^-_2 \leq a^-_3 \leq a^-_4, 0 \leq a^+_1 \leq a^+_2 \leq a^+_3 \leq a^+_4$, let $h^-_A$ and $h^+_A$ denote the heights of $A^-$ and $A^+$,

$0 \leq h^- \leq h^+ \leq 1$. The lower membership function $u^-(x)$ and the upper membership function $u^+(x)$ are defined as:

$$u^-(x) = \begin{cases} \frac{h^-_1(x-a^-_1)}{a^-_4-a^-_1}, & a^-_1 < x < a^-_4 \\ \frac{h^-_2(a^-_4-x)}{a^-_4-a^-_1}, & a^-_1 < x < a^-_4 \\ 0, & \text{otherwise} \end{cases}$$

(1)
Definition 2[8]: Let
\[ A_1 = \left\{ (a_{ij}^+, a_{ij}^-, a_{ij}, b_{ij}^+) \mid (a_{ij}^+, a_{ij}^-, a_{ij}, b_{ij}^+) \right\}, \]
\[ A_2 = \left\{ (a_{ij}^+, a_{ij}^-, a_{ij}, b_{ij}^+) \mid (a_{ij}^+, a_{ij}^-, a_{ij}, b_{ij}^+) \right\} \]

are two IT2rFSs, the addition operation and multiplication with real number are defined as follows:
\[ A_1 \oplus A_2 = \left\{ (a_{ij}^+, a_{ij}^-, a_{ij}, b_{ij}^+) \mid (a_{ij}^+, a_{ij}^-, a_{ij}, b_{ij}^+) \right\}, \]
\[ A_1 \otimes A_2 = \left\{ (a_{ij}^+, a_{ij}^-, a_{ij}, b_{ij}^+) \mid (a_{ij}^+, a_{ij}^-, a_{ij}, b_{ij}^+) \right\} \]

(2)Compute the compromise solution \( Q \):
\[ Q = \theta(S_j - S^* \cdot (S^* - S^*) + (1 - \theta)(R_j - R^*) / (R^* - R^*) \]
\[ S^* = \min S_j, S^- = \max S_j, \]
\[ R^* = \min R_j, R^- = \max R_j \]

Thus, we can obtain the ranking order according to the following two conditions:
\[ con_1: Q^{(2)} - Q^{(i)} \geq \frac{1}{m-1}, \text{ where } Q^{(i)} \text{ denote the minimum compromise solution, } Q^{(1)} \text{ is the second minimum compromise solution, and } m \text{ denotes the quantity of alternatives.} \]
\[ con_2: \text{The alternative } A^{(1)} \text{ must also with the minimum } S_j \text{ or/and } R_j, \text{where } A^{(1)} \text{ denote the alternative compromise solution.} \]

We can obtain a set of compromise solutions if the two conditions are not satisfied simultaneously,which consists of alternatives \( A^{(1)} \) and \( A^{(2)} \) if only the \( con_2 \) is not satisfied,or alternatives \( A^{(1)} \), \( A^{(2)} \),…….\( A^{(M)} \) if the \( con_1 \) is not satisfied; \( A^{(M)} \) is determined by the relation \( Q^{(M)} - Q^{(l)} < \frac{1}{m-1} \) for maximum \( M \).

III. VIKOR METHOD BASED ON INTERVAL TYPE-2 FUZZY SETS

Assume \( Z \) denotes the set of alternatives, \( Z = \{z_1, z_2, \ldots, z_n\} \), \( C \) denotes the set of attributes and \( W \) denotes the set of weights ,where \( C = \{c_1, c_2, \ldots, c_n\} \), \( W = \{w_1, w_2, \ldots, w_n\} \), and \( 0 \leq w_i \leq 1, \sum_{i=1}^{n} w_i = 1 \).The value \( A_j \) denotes the evaluation value of alternative \( Z \) under criterion \( C_j \), which is in the form of IT2TrFNs. \( u_{A_j}^+(x) \) and \( u_{A_j}^-(x) \) denote the upper membership function and the lower membership function of \( A_j \), respectively.
A. Ranking score value based on signed area

For multi attribute decision making (MADM) problems, where the evaluation value of each alternative under every attribute is IT2TrFNs, the key point to obtain the final rank order of all alternatives is to find an appropriate method to deal with the evaluation values. In this paper, an extended VIKOR method is present to solve the problem. A new signed area formulation is proposed based on definite integral, then, the ranking score value of interval type-2 fuzzy sets can be obtained by the proposed signed area formulation. Thus, we can get the final order of all alternatives combined with the VIKOR method.

Assume \( A = (A_1, A_2, \ldots, A_n) \) is a set of trapezoidal fuzzy numbers, where the membership function of \( A_i \) is \( u_i(x) \), \( 1 \leq i \leq n \). \( h(A) \) is the height of \( A_i \), \( 0 \leq h(A_i) \leq 1 \).

As shown in Fig. 1, We can compute the signed area \( S_L(A) \) and \( S_U(A) \) between \( u_i(x) \) and \( x = 0 \) as follows, while \( a_i \leq x \leq b_i \) and \( c_i \leq x \leq d_i \).

\[
S_L(A_i) = b_i \cdot h(A_i) - \int_0^{b_i} u_i(x) dx
\]

(10)

\[
S_U(A_i) = c_i \cdot h(A_i) + \int_0^{c_i} u_i(x) dx
\]

(11)

So, the ranking score value \( \text{score}(A) \) can be gained based on (10)-(11):

\[
\text{score}(A) = \left( \frac{S_L(A_i) + S_U(A_i)}{T(A)} \right) \cdot h(A_i), \quad 1 \leq i \leq n
\]

(12)

\( T(A) \) represents the area of rectangle, whose height is \( h(A_i) \) and the bottom edge is \( x_{max} \),

\[
T(A) = h(A_i) \cdot x_{max}, \quad 1 \leq i \leq n.
\]

With (10)-(11), we can rewrite (12) as:

\[
\text{score}(A) = \left( \frac{h(A_i) \cdot \int_0^{b_i} u_i(x) dx + c_i \cdot h(A_i) + \int_0^{c_i} u_i(x) dx}{h(A_i) \cdot x_{max}} \right) \cdot h(A_i)
\]

(13)

B. Decision making steps for Extended VIKOR based on IT2TrFNs

As is shown in Fig. 2, for interval type-2 trapezoidal fuzzy numbers, we can compute the the ranking score value \( \text{score}(A_i^U) \) and \( \text{score}(A_i^L) \) similarly as (13).

Thus, we can get the decision making steps for extended VIKOR based on IT2TrFNs:

Step 1: compute the signed area \( S_L(A_i^U) \) and \( S_U(A_i^U) \) between \( u_i^U(x) \) and \( x = 0 \), while \( a_i^U \leq x \leq b_i^U \) and \( c_i^U \leq x \leq d_i^U \); And compute the signed area \( S_L(A_i^L) \) and \( S_U(A_i^L) \) between \( u_i^L(x) \) and \( x = 0 \), while \( a_i^L \leq x \leq b_i^L \) and \( c_i^L \leq x \leq d_i^L \) (as shown in Fig. 2).

\[
S_L(A_i^U) = b_i^U \cdot h(A_i^U) - \int_{a_i^U}^{b_i^U} u_i^U(x) dx
\]

(14)

\[
S_U(A_i^U) = c_i^U \cdot h(A_i^U) + \int_{c_i^U}^{d_i^U} u_i^U(x) dx
\]

(15)

\[
S_L(A_i^L) = b_i^L \cdot h(A_i^L) - \int_{a_i^L}^{b_i^L} u_i^L(x) dx
\]

(16)

\[
S_U(A_i^L) = c_i^L \cdot h(A_i^L) + \int_{c_i^L}^{d_i^L} u_i^L(x) dx
\]

(17)

Where, \( 0 \leq h(A_i^U) \leq 1 \), \( 0 \leq h(A_i^L) \leq 1 \); \( u_i^U(x) \) and \( u_i^L(x) \) represent the upper membership function and the lower membership function of \( A_i \), respectively.
Step 2: Compute the ranking score value $score(A^u_i)$ based on $S_L(A^U_i)$ and $S_U(A^L_i)$:

$$score(A^u_i) = \frac{S_L(A^U_i) + S_U(A^L_i)}{2}, \quad 1 \leq i \leq m$$

$$T(A^u_i) = h(A^u_i) \cdot x_{max}, \quad 1 \leq i \leq m$$

Substitute (14) with (10)-(11), we can further get the ranking score value $score(A^u_i)$ as follows:

$$score(A^u_i) = \left[ \left( h(A^u_i) \cdot x_{max} \right)^2 \cdot \left( \frac{t(A^u_i)}{\sqrt{\sum h(A^u_i)}} \right)^2 \right]^{1/2} \frac{h(A^u_i)}{2}$$

Step 3: Similarly, we can obtain the ranking score value $score(A^l_i)$:

$$score(A^l_i) = \frac{score(A^u_i) + score(A^l_i)}{2}$$

Step 4: Compute the ranking score value $score(A^i_j)$:

$$score(A^i_j) = \frac{score(A^u_i) + score(A^l_i)}{2}, \quad i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$$

Step 5: Determine the best value and the worst value, respectively:

$$score(A^i_j)^+ = \max \{ score(A^i_j) \}, j \in H_b$$

$$score(A^i_j)^- = \min \{ score(A^i_j) \}, j \in H_c$$

Step 6: Compute the distance between $score(A^i_j)$ and $score(A^i_j)^+$, and the distance between $score(A^i_j)^-$ and $score(A^i_j)^+$:

$$d(score(A^i_j), score(A^i_j)^+) = score(A^i_j)^+ - score(A^i_j)$$

$$d(score(A^i_j)^-, score(A^i_j)^+) = score(A^i_j)^- - score(A^i_j)^+$$

Step 7: Use (6) - (9) to calculate group utility $S_i$, individual regret $R_i$ and compromise solution $Q_i$ and obtain the final order according.

IV. NUMERICAL EXAMPLE

(18) Assume that a firm intends to search for a project of venture capital, where four projects $A_1, A_2, A_3, A_4$ are available. Four attributes are considered to decide which project the firm should make an investment, $G_1$: substitutes of product, $G_2$: capacity and experience of management, $G_3$: technology of investment, and $G_4$: the rationality of capital structure. The weighting vector is $w = (0.3, 0.2, 0.1, 0.4)$. The firm uses the following linguistic terms to evaluate the four projects:

$S = \{ s_1 = \text{extremely poor}, s_2 = \text{very poor}, s_3 = \text{poor}, s_4 = \text{slightly poor}, s_5 = \text{fair}, s_6 = \text{slightly good}, s_7 = \text{good}, s_8 = \text{very good}, s_9 = \text{extremely good} \}$. Suppose the decision matrix $D$ is given by the expert as follows:

$$D = \begin{bmatrix}
    d_{11} & d_{12} & d_{13} & d_{14} \\
    d_{21} & d_{22} & d_{23} & d_{24} \\
    d_{31} & d_{32} & d_{33} & d_{34} \\
    d_{41} & d_{42} & d_{43} & d_{44}
\end{bmatrix}$$

Based on the method of [9], we can obtain $D$ in the form of interval type-2 trapezoidal fuzzy sets:

$$D = \begin{bmatrix}
    d_{11} & d_{12} & d_{13} & d_{14} \\
    d_{21} & d_{22} & d_{23} & d_{24} \\
    d_{31} & d_{32} & d_{33} & d_{34} \\
    d_{41} & d_{42} & d_{43} & d_{44}
\end{bmatrix}$$

Step 1: Compute the signed area $S_L(A^U_j), S_U(A^L_j)$ and $S_L(A^L_j), S_U(A^U_j)$:

$$S_L(A^U_j) = \begin{bmatrix}
    5/2 & 4 & 5 & 11/2 \\
    9/2 & 11/2 & 9/2 & 11/2 \\
    11/2 & 9/2 & 13/2 & 4 \\
    7/2 & 9/2 & 7/2 & 13/2
\end{bmatrix}$$

$$S_U(A^L_j) = \begin{bmatrix}
    5/2 & 4 & 5 & 11/2 \\
    9/2 & 11/2 & 9/2 & 11/2 \\
    11/2 & 9/2 & 13/2 & 4 \\
    7/2 & 9/2 & 7/2 & 13/2
\end{bmatrix}$$

$$S_L(A^L_j) = \begin{bmatrix}
    11/2 & 13/2 & 17/2 & 8 \\
    17/2 & 15/2 & 13/2 & 17/2 \\
    8 & 17/2 & 17/2 & 13/2 \\
    6 & 15/2 & 11/2 & 17/2
\end{bmatrix}$$

$$S_U(A^U_j) = \begin{bmatrix}
    11/2 & 13/2 & 17/2 & 8 \\
    17/2 & 15/2 & 13/2 & 17/2 \\
    8 & 17/2 & 17/2 & 13/2 \\
    6 & 15/2 & 11/2 & 17/2
\end{bmatrix}$$
Step 2: Compute the matrix of ranking score value \( \text{score}(A_j) \) based on signed area \( S_U(A_j^U), S_U(A_j^L) \), and \( S_L(A_j^L) \) :

\[
\text{score}(A_j) = \begin{bmatrix}
0.44 & 0.58 & 0.75 & 0.75 \\
0.72 & 0.72 & 0.61 & 0.78 \\
0.75 & 0.72 & 0.83 & 0.58 \\
0.53 & 0.67 & 0.5 & 0.83
\end{bmatrix}
\]

Step 3: Determine the ideal \( \text{score}(A_j)^{+} \) and the negative \( \text{score}(A_j)^{-} \) of all criteria ratings \( j = 1, 2, \ldots, n \) and alternatives \( i = 1, 2, \ldots, m \) :

\[
\text{score}(A_j)^{+} = (0.75, 0.72, 0.83, 0.83) \\
\text{score}(A_j)^{-} = (0.44, 0.58, 0.5, 0.58)
\]

Step 4: Compute the group utility \( S_i \), individual regret \( R_i \), and compromise solution \( Q_i \) :

\[
S_i = 0.65, S_2 = 0.18, S_3 = 0.40, S_4 = 0.38 \\
R_1 = 0.3, R_2 = 0.08, R_3 = 0.4, R_4 = 0.21 \\
Q_1 = 0.84, Q_2 = 0, Q_3 = 0.78, Q_4 = 0.45
\]

Step 5: Obtain the final order according to the condition \( \text{con}_1 \) and \( \text{con}_2 \) :

\[
Q_4 - Q_2 = 0.45 \geq \frac{1}{3}, \text{which is satisfied the con}_1; \\
\text{the minimum compromise solution is } Q_2 = 0, \text{where the minimum } S_i \text{ and } R_i \text{ are } S_2 = 0.18 \text{ and } R_2 = 0.08, \text{ which is satisfied the con}_2. \text{So, the rank of these four projects is shown as follows: } Q_2 > Q_4 > Q_3 > Q_1.
\]

V. CONCLUSION

In this paper, a new MADM method is proposed combined with VIKOR method, which can directly deal with decision making attributes given in the form of interval type-2 fuzzy sets. A new signed area function with trapezoidal fuzzy number is defined and a new ranking score method is presented consequently. The proposed method shows its effectiveness via a numerical example and potential for more real decision making problems in the future.

REFERENCES