Effective Control for Wireless Sensor and Mobile Actuator Network in Regulation of Environmental Density Function

Mu-Tai Lin, Tseng-Chang Lin and Yen-Chen Liu

Networked Robotic System Laboratory
Department of Mechanical Engineering
National Cheng Kung University

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Wireless Sensor Networks (WSN)

- Monitoring, information gathering\(^1\)

Hazardous Environments\(^2\)

Battlespace\(^3\)

Biomedical\(^4\)


Wireless Sensor & Actuator Networks (WSAN)

- Monitoring, information gathering, **control output**
Multi-Robot Coverage Control\textsuperscript{6}

- Find optimal distribution according to importance density.
- Drive mobile robots to cover higher density region.

Motivation

• Combine multi-robot control system with WSAN
System Formulation

- Two dimensional convex set: $Q \in \mathbb{R}^2$
- Arbitrary points: $q \in Q$
- Sensing density function: $\phi(q, t) \in \mathbb{R}$

State space equation

$$\dot{x} = A(t)x(t) + B(t)u(t)$$

- $A(t)$: influence of $q$ by other points in environment
- $B(t)$: influence of $q$ by actuator on mobile robots
- $u(t)$: influence of actuators on mobile robots affecting the environment, denoting as $u^a(t)$ in the following

Mobile robots control law:

$$\dot{p}_i = u_i$$

- $p_i$: position of the $i^{th}$ mobile robot
Environment Model Construction

- Sensing Density Function: \( \phi(q) \)
- Estimated Density Function: \( \hat{\phi}(q) = a\psi(\mu, q) \)
- Gaussian Mixture Model: \( \psi(\mu, q) = \sum_{j=1}^{n_b} \omega_j e^{-\frac{||q-\mu_j||^2}{2\sigma^2}} \)
Environment Model Construction

EM algorithm
Gaussian basis:

\[ g(\|q - \mu_j\|) = e^{-\frac{\|q - \mu_j\|^2}{2\sigma^2}} \]

Latent variable (E-Step):

\[ z_j^k = \frac{g(\|s_j - \mu_i\|)\phi(s_j)}{\psi(\mu, s_j)} \]

Calculated Center of Gaussian (M-step):

\[ \mu_j = \frac{\sum_{k=1}^{n_s} S_k Z_j^k}{\sum_{k=1}^{n_s} Z_j^k} \]
Multi-Robot Coverage Control

Voronoi Partition

\[ V_i = \{ q \in Q \mid \| q - p_i \| \leq \| q - p_j \|, \quad \forall j \neq i \} \]

- Shortest distance information
- Neighbor Definition
- Collision Avoidance

Robot Dynamic

\[ \dot{p}_i = u_i \]
Multi-Robot Coverage Control

Cost Function

\[ \mathcal{H}(P) = \frac{1}{2} \sum_{i=1}^{n} \int_{V_i} \| q - p_i \|^2 \varepsilon^2(q, t) dq \]

where \( \varepsilon = \begin{cases} \hat{\phi}(q, t) - R, & \text{for } \hat{\phi}(q, t) > R \\ 0, & \text{for } \hat{\phi}(q, t) \leq R \end{cases} \)

Voronoi Mass, Momentum, and Centroid

\[ \hat{M}_{V_i} = \int_{V_i} \psi(\mu, q) dq, \quad \hat{L}_{V_i} = \int_{V_i} q \psi(\mu, q) dq, \quad \hat{C}_{V_i} = \frac{\hat{L}_{V_i}}{\hat{M}_{V_i}} \]

Coverage Controller

\[ u_i = -K(p_i - \hat{C}_{V_i}) \]
Latent Transform Method

Use for other model more than

\[ f(\|q - p_i\|) = \frac{1}{2} \|q - p_i\|^2 \]

EM algorithm for finding optimal solution

Performance Function

\[ J_p = \sum_{i=1}^{n_a} \int_Q f(\|q - p_i\|)\varphi(q) dq \]

Sensor Model

\[ \psi(p, q) = \frac{1}{n_a} \sum_{i=1}^{n_a} f(\|q - p_i\|) \]
Latent Transform Method

Performance Function

\[ J_p = \sum_{i=1}^{n_a} \int_Q f(||q - p_i||)\varphi(q) dq \]

\[ = n_a \int_Q \psi(p, q)\varphi(q) dq \]

Ln-likelihood Function

\[ L = \int_Q \ln \psi(p, q)\varphi(q) dq \]

Equivalence

\[ \arg \max J_p = \arg \max L \]
Latent Transform Method

Expectation step

\[ z_i(q) = \frac{f(\|q - p_i\|)\varphi(q)}{\psi(p, q)} \]

Maximization step

\[ \mu_i = \frac{\int_Q qz_i(q) dq}{\int_Q z_i(q) dq} \]

P-controller

\[ u_{i,EM} = -K_p(p_i - \mu_i) \]
Collaborative Actuator Control

- Define the neighbor $N_i$ of each mobile robots whose actuating range is $2\sigma$

$$N_i = \{ p_j \mid \|p_i - p_j\| < 2\sigma, \forall j \neq i \}$$

- If distance of two robots is less enough to generate the overlapping, define the number of nearby robots of $i$ be $N_i = \dim(N_i)$

- Actuator output

$$u_i^a = \frac{1}{n_i + 1} \tilde{u}_i^a$$

where $\tilde{u}_i^a$ is the original output of actuator on mobile robots affecting the environment
Simulation Result

- Initial condition
Simulation Result

- Voronoi partition and Lloyd controller
Simulation Result

- Latent transform method

Estimated Density Function

Lloyd Method Trajectories
where error is the difference between the sensing density function and the reference
Conclusion

- We propose a system framework that combines multi-robot control system with WSAN by embedding sensor and actuator in mobile robots.
- We propose a Latent transform method for more kinds of mobile robots sensing models.
- Utilize Latent transform and EM algorithm to coverage the estimate sensing density function constructed from Gaussian mixture model, and obtain a better coverage result.
Thank you for your listening

Mu-Tai Lin
n16054954@mail.ncku.edu.tw
Networked Robotic System Laboratory
Department of Mechanical Engineering
National Cheng Kung University