Project Title: Controller Design based on Root Contour for Non-minimum Phase UAV System

Abstract—This paper aims to analyze the scale model of the Cessna 182 and design the controller that meets the mission requirements. Different from other references that use Ziegler–Nichols method or another trial and error method to design the PID controller, this research applies the Root Contour method (RC) with multiple variables to certainly reflect the changes and relationships between the controller parameters and help design the better controller to improve the system performance and accuracy. In addition, face of the instability effect due to the non-minimum phase system from the transfer function between the speed and the elevator angle, this paper proposes a new design method of controller with the reverse multi-variable RC. The new method is practical and efficient to design the controllers and allow the operators can find the appropriate controller parameters more quickly and obtain the better system performances.

Research Methods or Steps:

A. PID controller based on RL criterion:

\[ u_{PID}(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt} \]

- \( K_p \): Proportional gain
- \( K_i \): Integral gain
- \( K_d \): Derivative gain
- \( e \): Error
- \( t \): Time

B. Ziegler–Nichols Method:

<table>
<thead>
<tr>
<th>Controller</th>
<th>( K_p )</th>
<th>( K_i )</th>
<th>( K_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>0.5K_u</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>PI</td>
<td>0.45K_u</td>
<td>0.45K_u</td>
<td>0</td>
</tr>
<tr>
<td>PID</td>
<td>0.6K_u</td>
<td>0.6K_u/0.5T_u</td>
<td>0.6K_u/0.125T_u</td>
</tr>
</tbody>
</table>

\( K_u \): maximum proportional
\( T_u \): Oscillation period

C. Root Contours Method:

Characteristic polynomial

\[ Q(s) + K_1 R_1(s) + K_2 R_2(s) = 0, \]

\( K_1 \) and \( K_2 \) are the variable (1)

Set any one of the variable is zero.

Step 1: Here we set variable \( K_2 \) is 0 first. Then \( Q(s) + K_1 R_1(s) = 0 \), draw the RL.

Step 2: In Eq. (1), we set \( K_1 \) is a random constant, and \( K_2 \) is the system variable, then draw the RL.

Step 3: Repeat step 2 several times and combine total RL to form the RC.

Research results:

A. Elevator angle to angle of attack:

\[ C_1(s) = \frac{0.29 + (s^2 + 6.9s + 51.7)}{s} \]

B. Speed to elevator angle:

\[ C_2(s) = \frac{0.29 + (s^2 + 8.9s + 41)}{s} \]

C. Reverse gain PID controller:

\[ C_3(s) = -0.01805 \times (s^3 + 2.495s^2 + 0.1255) \]

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